

Nonlinear spin diffusion and spin rotation in a trapped Fermi gas

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Transverse spin diffusion in a polarized, interacting Fermi gas leads to the Leggett-Rice effect, where the spin current precesses around the local magnetization. With a spin-echo sequence both the transverse diffusivity and the spin-rotation parameter γ are obtained; the sign of γ reveals the repulsive or attractive character of the effective interaction. In a trapped Fermi gas the spin diffusion equations become nonlinear, and their numerical solution exhibits an inhomogeneous spin state even at the spin echo time. While the microscopic diffusivity and γ increase at weak coupling, their apparent values inferred from the trap-averaged magnetization saturate in agreement with a recent experiment for a dilute ultracold Fermi gas.

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I. INTRODUCTION

Transverse spin diffusion occurs when the magnetization is oriented along different directions, for instance, in a spin helix. It has been observed in spin-echo experiments in helium [1], polarized hydrogen, and, recently, ultracold atomic gases [2–4]. The transverse magnetization evolves according to a diffusion equation, but there are additional terms from the precession of the spin currents around the local magnetization. This Leggett-Rice effect [5] is related to the identical spin-rotation effect [6] and leads to reactive spin currents and spin waves, which have been observed in ultracold Fermi gases [7]. Consider a polarized sample with small transverse magnetization (small tipping angle). The transverse magnetization is conveniently combined into a complex number $m_+ = m_x + im_y$, which evolves with a complex diffusion coefficient,

$$\frac{\partial m_+}{\partial t} = \frac{D_0^\perp}{1 - i\mu m_z} \nabla^2 m_+ - i\alpha x_3 m_+, \quad (1)$$

where D_0^\perp is the transverse diffusivity and $\mu m_z = \gamma$ (at full polarization) the dimensionless spin-rotation parameter. For small $\mu \rightarrow 0$ this is the usual diffusion equation, while for large $|\mu|$ the diffusion equation has an imaginary effective diffusivity and resembles the Schrödinger equation [8]. In a spin-echo pulse sequence, the second term in Eq. (1) expresses a linear gradient α of the external magnetic field along the x_3 direction, which winds the magnetization into a helix. After a time t_π , a π pulse is applied, which is equivalent to flipping the sign of α . In the ensuing time evolution, the helix unwinds until the magnetization is realigned at the echo time $t_e = 2t_\pi$. In the presence of spin rotation $\mu \neq 0$, realignment at t_e occurs at a phase angle $\phi \propto \mu$ with respect to the initial orientation at $t = 0$ [5], and the value of μ can be inferred. For a strongly interacting Fermi gas the value of μ has recently been measured and used to determine the spin-antisymmetric Fermi-liquid parameter F_1^a , while the sign of μ reveals the attractive or repulsive character of the effective interaction [4]. Theoretically, D_0^\perp and μ

for dilute, homogeneous Fermi gases have been computed using kinetic theory [9, 10].

In an infinite homogeneous system where the only position dependence arises from the magnetic field gradient α , the phase angle ϕ is directly proportional to the microscopic parameter μ . Instead, for a finite homogeneous box the phase ϕ , and hence the apparent value of μ , saturates when the system size is reached [11]. Experiments with ultracold atomic gases typically employ a harmonic trapping potential which is both finite and inhomogeneous: in this case, the diffusivity D_0^\perp , the Leggett-Rice parameter μ , and the magnetization m_z are strongly position dependent, and the diffusion equation (1) becomes nonlinear. In previous studies the evolution has been linearized in order to determine the collective mode frequencies and decay rates in the trapping potential [12]. Other studies consider the nonlinear evolution of the full phase-space distribution for a nondegenerate or collisionless trapped gas [13]. Here, I numerically solve the nonlinear evolution equation with position dependent kinetic coefficients including medium scattering to obtain the transverse magnetization decay and the growth of the phase ϕ for the specific trap geometry used in the experiment.

Kinetic theory [14] is employed to compute the spin evolution of the trapped gas. This method is well controlled in the weak-coupling limit, which is also the parameter regime where finite-size corrections due to the trapping potential are most pronounced [4]. On the other hand, at strong coupling toward unitarity and at low temperature near the superfluid phase transition of an attractive Fermi gas [15], kinetic theory is expected to receive quantitative corrections from the effects of pairing and short quasiparticle lifetimes, which are incorporated for instance when computing transport from the Kubo formula within a Luttinger-Ward approach [16, 17].

This paper is structured as follows: in Sec. II the spin evolution equations for a trapped Fermi gas are derived from kinetic theory, while Sec. III compares them to the known homogeneous limit. Section IV presents the results for the magnetization profiles and apparent diffusivities, and Sec. V concludes with a discussion.

II. KINETIC THEORY

Transport in an interacting Fermi gas may be described by kinetic theory for quasiparticles, as long as they are sufficiently long-lived. For a multi-component Fermi gas with two or more spin species one has to compute the time evolution of the spin distribution \underline{n}_p , which is a matrix with components $\underline{n}_p = n_{p\sigma\sigma'}(\mathbf{x}, t)$ in spin space. The evolution equation derived by Landau and Silin [18, 19] reads

$$\frac{\partial \underline{n}_p}{\partial t} + \frac{1}{2}[\nabla_p \underline{\varepsilon}_p, \nabla_r \underline{n}_p]_+ - \frac{1}{2}[\nabla_r \underline{\varepsilon}_p, \nabla_p \underline{n}_p]_+ + \frac{i}{\hbar}[\underline{\varepsilon}_p, \underline{n}_p]_- = \left(\frac{\partial \underline{n}_p}{\partial t} \right)_{\text{coll}} \quad (2)$$

where $\underline{\varepsilon}_p = \varepsilon_{p\sigma\sigma'}(\mathbf{x}, t)$ is the matrix of single-particle energies. The left-hand side constitutes the drift term, while the right-hand side describes the change in the distribution caused by collisions. Specifically for the spin-1/2 case, \underline{n}_p and $\underline{\varepsilon}_p$ are 2×2 matrices in spin space, for instance, in the \uparrow, \downarrow basis.

The spin matrices can be decomposed in terms of the identity \underline{I} and the Pauli matrices $\underline{\sigma}$. The occupation number matrix is written as

$$\underline{n}_p = \frac{1}{2}(f_p \underline{I} + \boldsymbol{\sigma}_p \cdot \underline{\sigma}) \quad (3)$$

where $f_p(\mathbf{x}, t)$ is the particle number distribution function and $\boldsymbol{\sigma}_p(\mathbf{x}, t)$ the spin vector distribution in Bloch space. Similarly, the energy matrix

$$\underline{\varepsilon}_p = \varepsilon_p \underline{I} + \mathbf{h}_p \cdot \underline{\sigma} \quad (4)$$

combines the spin-independent single-particle energies $\varepsilon_p(\mathbf{x}, t)$ and a magnetic field $\mathbf{h}_p(\mathbf{x}, t)$. One may then rewrite Eq. (2) as

$$\frac{\partial f_p}{\partial t} + \sum_j \left[\frac{\partial \varepsilon_p}{\partial p_j} \frac{\partial f_p}{\partial x_j} - \frac{\partial \varepsilon_p}{\partial x_j} \frac{\partial f_p}{\partial p_j} + \frac{\partial \mathbf{h}_p}{\partial p_j} \cdot \frac{\partial \boldsymbol{\sigma}_p}{\partial x_j} - \frac{\partial \mathbf{h}_p}{\partial x_j} \cdot \frac{\partial \boldsymbol{\sigma}_p}{\partial p_j} \right] = \left(\frac{\partial f_p}{\partial t} \right)_{\text{coll}} \quad (5)$$

and

$$\frac{\partial \boldsymbol{\sigma}_p}{\partial t} + \sum_j \left[\frac{\partial \varepsilon_p}{\partial p_j} \frac{\partial \boldsymbol{\sigma}_p}{\partial x_j} - \frac{\partial \varepsilon_p}{\partial x_j} \frac{\partial \boldsymbol{\sigma}_p}{\partial p_j} + \frac{\partial \mathbf{h}_p}{\partial p_j} \frac{\partial f_p}{\partial x_j} - \frac{\partial \mathbf{h}_p}{\partial x_j} \frac{\partial f_p}{\partial p_j} \right] - \frac{2}{\hbar} \mathbf{h}_p \times \boldsymbol{\sigma}_p = \left(\frac{\partial \boldsymbol{\sigma}_p}{\partial t} \right)_{\text{coll}}. \quad (6)$$

The spin-rotation term $\mathbf{h}_p \times \boldsymbol{\sigma}_p$ is responsible for the Leggett-Rice effect. The single-particle energies are

$$\varepsilon_p(\mathbf{x}, t) = \frac{p^2}{2m} + V(\mathbf{x}), \quad V(\mathbf{x}) = \frac{m}{2} \sum_j \omega_j^2 x_j^2 \quad (7)$$

for a Fermi gas in a harmonic trapping potential $V(\mathbf{x})$, which can be anisotropic with different trapping frequencies ω_j in spatial direction j . The magnetic field

$$\mathbf{h}_p(\mathbf{x}, t) = -\frac{\hbar}{2} \boldsymbol{\Omega}(\mathbf{x}, t), \quad \boldsymbol{\Omega} = \boldsymbol{\Omega}_0 + \boldsymbol{\Omega}_{\text{mf}}, \quad (8)$$

$$\boldsymbol{\Omega}_0(\mathbf{x}, t) = \alpha(t) x_3 \hat{\mathbf{z}}, \quad \boldsymbol{\Omega}_{\text{mf}} = \frac{W}{\hbar} \mathbf{m}(\mathbf{x}, t) \quad (9)$$

is written in terms of the Larmor frequency $\boldsymbol{\Omega} = \boldsymbol{\Omega}_0 + \boldsymbol{\Omega}_{\text{mf}}$. For a spin-echo protocol it has two contributions: (i) $\boldsymbol{\Omega}_0 = \gamma \mathbf{B}(\mathbf{x}, t)$ is due to the external magnetic field $\mathbf{B}(\mathbf{x}, t)$ with gyromagnetic ratio γ . A spatially constant \mathbf{B} is compensated by going to the co-rotating frame in Bloch space, but a magnetic field B_z gradient of slope α along the x_3 direction winds up the local magnetization into a spin spiral. (ii) The second contribution to the Larmor frequency, $\boldsymbol{\Omega}_{\text{mf}}$, is a mean-field term proportional to the local magnetization \mathbf{m} of a polarized Fermi gas. It leads to the precession of the spin current around \mathbf{m} .

The evolution of the full distribution functions $f_p(\mathbf{x}, t)$ and $\boldsymbol{\sigma}_p(\mathbf{x}, t)$ is simplified by considering moments with respect to momentum:

$$n(\mathbf{x}, t) = \int \frac{d^3 p}{(2\pi\hbar)^3} f_p(\mathbf{x}, t) \quad (10)$$

$$J_j^n(\mathbf{x}, t) = \int \frac{d^3 p}{(2\pi\hbar)^3} \frac{\partial \varepsilon_p}{\partial p_j} f_p(\mathbf{x}, t) \quad (11)$$

$$\mathbf{m}(\mathbf{x}, t) = \int \frac{d^3 p}{(2\pi\hbar)^3} \boldsymbol{\sigma}_p(\mathbf{x}, t) \quad (12)$$

$$\mathbf{J}_j(\mathbf{x}, t) = \int \frac{d^3 p}{(2\pi\hbar)^3} \frac{\partial \varepsilon_p}{\partial p_j} \boldsymbol{\sigma}_p(\mathbf{x}, t) \quad (13)$$

with bare velocity $v_{pj} = \partial \varepsilon_p / \partial p_j = p_j / m$. An additional contribution $f_p \partial \mathbf{h}_p / \partial p_j$ to the spin current is absent for momentum-independent \mathbf{h}_p . The local polarization is defined as $\mathbf{M}(\mathbf{x}, t) = \mathbf{m}(\mathbf{x}, t) / n(\mathbf{x}, t)$ where $|\mathbf{M}(\mathbf{x}, t)| \leq 1$. The spin current \mathbf{J}_j is both a vector in Bloch space (bold symbol) and a vector in position space (j index): it encodes how the magnetization changes as one goes along the j direction. The evolution equations for the moments read, using the specific form of the single-particle energies (7)–(9),

$$\partial_t n + \sum_j \nabla_j J_j^n = 0 \quad (14)$$

$$\partial_t J_j^n + \alpha_n \nabla_j n + \omega_j^2 x_j n = -\frac{J_j^n}{\tau_n} \quad (15)$$

$$\partial_t \mathbf{m} + \sum_j \nabla_j \mathbf{J}_j + \boldsymbol{\Omega}_0 \times \mathbf{m} = 0 \quad (16)$$

$$\partial_t \mathbf{J}_j + \alpha_{\parallel} P_{\parallel} \nabla_j \mathbf{m} + \alpha_{\perp} P_{\perp} \nabla_j \mathbf{m} + \omega_j^2 x_j \mathbf{m} + \left(\boldsymbol{\Omega}_0 + \frac{W}{\hbar} \mathbf{m} \right) \times \mathbf{J}_j = \left(\frac{\partial \mathbf{J}_j}{\partial t} \right)_{\text{coll}}. \quad (17)$$

The projectors $P_{\parallel} \mathbf{a} \equiv (\mathbf{a} \cdot \hat{\mathbf{m}}) \hat{\mathbf{m}}$ and $P_{\perp} \equiv 1 - P_{\parallel}$ give the component of the magnetization gradient parallel and perpendicular to the local magnetization, respectively.

When deriving these equations, the mean field Ω_{mf} has been retained only in the spin-rotation term and not in the Poisson brackets in Eqs. (5) and (6). This can be justified as the leading order in a controlled large- N expansion [20]. In general, the evolution of the currents depends on the second moments of f_p and σ_p which in turn depend on higher moments. However, near local equilibrium the linearized Boltzmann equation relates the higher moments to the lower ones via the coefficients α_n , τ_n , $\alpha_{\parallel,\perp}$, τ_D , and W , and one obtains a closed set of evolution equations. These coefficients are discussed in Sec. II B.

A. Initial conditions

In the absence of the external magnetic field gradient α , the local equilibrium Fermi distribution is

$$n_{p\pm}(\mathbf{x}) = \frac{1}{\exp(\beta(\varepsilon_p(\mathbf{x}) - \mu_{\pm})) + 1} \quad (18)$$

in terms of the chemical potential μ_{\pm} of the majority (minority) component. The resulting density profile of the fully polarized gas is

$$n(\mathbf{x}) = -\lambda^{-3} \text{Li}_{3/2}(-z_+ e^{-\beta V(\mathbf{x})}) \quad (19)$$

with thermal wavelength $\lambda = (2\pi\hbar^2\beta/m)^{1/2}$ and polylogarithm $\text{Li}_s(z)$. The fugacity $z_+ = \exp(\beta\mu_+)$ of the majority component is determined by the total particle number N in the trap of average frequency $\bar{\omega} = (\omega_1\omega_2\omega_3)^{1/3}$,

$$N = -\frac{1}{(\beta\hbar\bar{\omega})^3} \text{Li}_3(-z_+). \quad (20)$$

At high temperatures in the Boltzmann regime the density profile is Gaussian, $n(\mathbf{x}) \propto \exp(-\beta V(\mathbf{x}))$, while at low temperatures it approaches the Thomas-Fermi profile,

$$n(\mathbf{x}) = n_0 \left(1 - \sum_j \frac{x_j^2}{R_{\text{TF}j}^2}\right)^{3/2} \quad (21)$$

with Thomas-Fermi radius $R_{\text{TF}j} = (2E_F/m\omega_j^2)^{1/2}$ and the Fermi energy $E_F = (6N)^{1/3}\hbar\bar{\omega}$ of a fully polarized gas. The density profile is shown in Fig. 1 both for weak coupling on the BCS side (upper panel) and for strong coupling at unitarity (lower panel). Note that the phase-space density $\lambda^3 n$ (solid black line) is unaffected by s -wave scattering in the fully polarized gas.

In the spin-echo protocol [4], the gas is initially fully polarized, $|\mathbf{m}| = n$ or $|\mathbf{M}| = 1$, at a tipping angle θ from the z axis in Bloch space,

$$\mathbf{m}(\mathbf{x}, t=0) = (\sin \theta, 0, \cos \theta)n(\mathbf{x}). \quad (22)$$

This distribution is stationary for $\alpha = 0$ and the particle and spin currents J_j^n and \mathbf{J}_j vanish. When the external gradient α is switched on along the x_3 direction, the

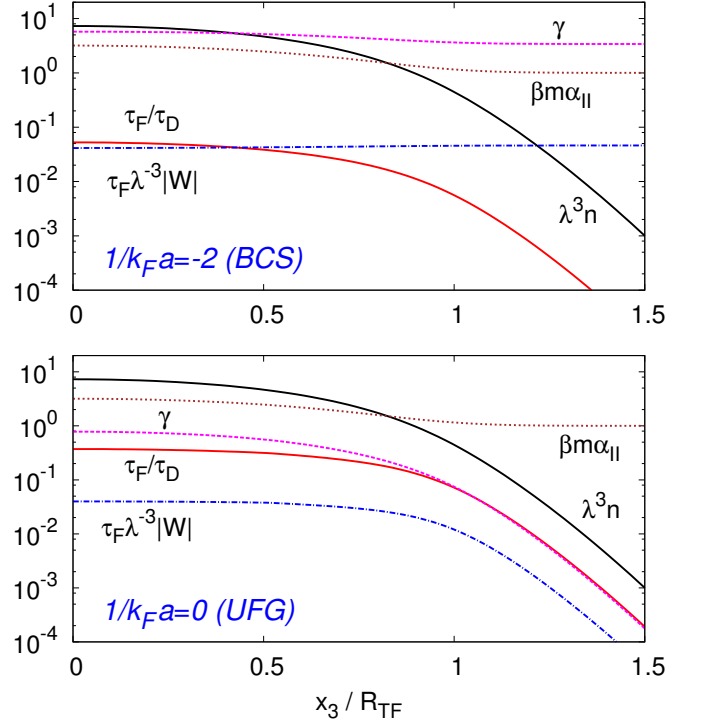


FIG. 1. (Color online) Kinetic coefficients and density profiles in the trap on the BCS side (upper panel) and at unitarity (lower panel) at degenerate temperature $(T/T_F)_i = 0.2$: total phase-space density $\lambda^3 n(\mathbf{x})$ (solid black line), diffusion parameter $\beta m \alpha_{\parallel}$ [dotted (brown) line], diffusive scattering rate τ_F/τ_D [solid (red) line], interaction parameter $\tau_F \lambda^{-3} |W|$ [dash-dotted (blue) line], and Leggett-Rice parameter $\gamma = -\tau_D W n/\hbar$ [dashed (magenta) line].

distribution remains independent of x_1 and x_2 , but currents are generated which can change the distribution along the x_3 direction. Hence, the spin evolution effectively reduces to a one-dimensional problem for \mathbf{m} and \mathbf{J}_3 along the gradient direction.

B. Kinetic coefficients

The coefficients $\alpha_{\parallel,\perp}(\mathbf{x})$ in Eq. (17) parametrize the strength of the spin current generated by a magnetization gradient. Their values are determined by the Boltzmann equation linearized around the local equilibrium solution [9, 10]. There are different contributions from the longitudinal magnetization gradient due to the trap potential, and the transverse magnetization gradient from the helix. For a fully polarized gas,

$$\alpha_{\parallel}(\mathbf{x}) = \alpha_n(\mathbf{x}) = \frac{n(\mathbf{x})}{m\chi(\mathbf{x})} \quad (23)$$

$$\alpha_{\perp}(\mathbf{x}) = \frac{P(\mathbf{x})}{mn(\mathbf{x})} \quad (24)$$

where $\chi(\mathbf{x}) = -\lambda^{-3}\beta \text{Li}_{1/2}(-z_+ e^{-\beta V(\mathbf{x})})$ is the local susceptibility, and $P(\mathbf{x}) = -\lambda^{-3}\beta^{-1} \text{Li}_{5/2}(-z_+ e^{-\beta V(\mathbf{x})})$ the

local pressure of a free Fermi gas. In Fig. 1, α_{\parallel} is plotted as the dotted line: for a high temperature or low density in the outer regions of the trap it reaches the Boltzmann limit $\alpha_{\parallel} = \alpha_{\perp} = 1/\beta m$, while for a high density at the trap center it is enhanced and approaches the low-temperature limit $\alpha_{\parallel} = v_F^2/3$ [5]. In general, α_{\parallel} and α_{\perp} acquire interaction corrections [9], but for large initial polarization and s -wave scattering these are negligible.

Second, the coefficients $\tau_{\parallel}(\mathbf{x})$ and $\tau_{\perp}(\mathbf{x})$ parametrize the decay of the spin current due to scattering [9],

$$\left(\frac{\partial \mathbf{J}_j}{\partial t}\right)_{\text{coll}} = \int \frac{d^3 p}{(2\pi\hbar)^3} \frac{\partial \varepsilon_p}{\partial p_j} \left(\frac{\partial \boldsymbol{\sigma}_p}{\partial t}\right)_{\text{coll}} = -\frac{P_{\parallel} \mathbf{J}_j}{\tau_{\parallel}} - \frac{P_{\perp} \mathbf{J}_j}{\tau_{\perp}}. \quad (25)$$

The component of the current $P_{\parallel} \mathbf{J}_j$ parallel to the local magnetization \mathbf{m} decays with the longitudinal diffusion time τ_{\parallel} , while the transverse component $P_{\perp} \mathbf{J}_j = (1 - P_{\parallel}) \mathbf{J}_j$ decays with the transverse diffusion time τ_{\perp} . Both differ for a polarized, strongly degenerate Fermi gas [9, 10, 21]: all the states between the majority and the minority Fermi surfaces of a polarized gas are available for transverse scattering and τ_{\perp} can be much lower than τ_{\parallel} . For the experiment at hand [4], however, the lowest temperatures reached at the trap center are around $T/T_F \gtrsim 0.3$, and previous studies have shown that in this temperature range τ_{\parallel} and τ_{\perp} are nearly equal [10]. It is therefore justified to work with a single decay time $\tau_D = \tau_{\perp} \approx \tau_{\parallel}$ [10],

$$\begin{aligned} \frac{\hbar}{\tau_{\perp}} &= \frac{\sinh(\beta\hbar)}{C_{\perp}} \frac{1}{(2\pi\hbar)^8} \int d^3 p_1 \dots d^3 p_4 \\ &\times \delta^{(3)}(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4) \delta(\varepsilon_{p_1} + \varepsilon_{p_2} - \varepsilon_{p_3} - \varepsilon_{p_4}) \\ &\times |\mathcal{T}(\mathbf{p}_1, \mathbf{p}_2)|^2 [e^{-\beta\hbar} n_{1+} n_{2+} + e^{\beta\hbar} n_{1-} n_{2-}] \\ &\times (1 - n_{3+})(1 - n_{4-}) v_{1j}(v_{1j} - v_{2j}) \end{aligned} \quad (26)$$

with magnetic field $\hbar = (\mu_+ - \mu_-)/2$ and normalization constant

$$C_{\perp} = \int \frac{d^3 p}{(2\pi\hbar)^3} v_{pj}^2 (n_{p+} - n_{p-}). \quad (27)$$

The T matrix $\mathcal{T}(\mathbf{p}_1, \mathbf{p}_2)$ describes s -wave scattering between particle $(\mathbf{p}_1, +)$ and particle $(\mathbf{p}_2, -)$. In order to derive explicit expressions, the T matrix for ultracold fermions with s -wave contact interactions has to be used. The two-body T matrix

$$\mathcal{T}_0(\mathbf{p}_1, \mathbf{p}_2) = \frac{4\pi\hbar^2}{m} \frac{1}{a^{-1} + ik} \quad (28)$$

is given in terms of the relative wavenumber $k = |\mathbf{p}_1 - \mathbf{p}_2|/2\hbar$ and the s -wave scattering length a . At weak coupling $|a| \rightarrow 0$, $\mathcal{T}_0(\mathbf{p}_1, \mathbf{p}_2) \rightarrow 4\pi\hbar^2 a/m$ is the regularized bare contact interaction. The BCS-BEC crossover goes from the weakly attractive BCS regime ($1/k_F a \lesssim -1$) via

the unitary Fermi gas (UFG, $1/k_F a = 0$) to the repulsive fermion branch ($1/k_F a \gtrsim 1$) above the BEC ground state [15]. At strong coupling the many-body T matrix is needed, which includes medium scattering. In the ladder approximation, which is the leading order of the large- N expansion in the number of fermion flavors [20, 22], the full T matrix reads

$$\begin{aligned} \mathcal{T}^{-1}(\mathbf{p}_1, \mathbf{p}_2) &= \mathcal{T}_0^{-1}(\mathbf{p}_1, \mathbf{p}_2) \\ &+ \int \frac{d^3 p}{(2\pi\hbar)^3} \frac{n_{\mathbf{p},+} + n_{\mathbf{p}+\mathbf{p}_1+\mathbf{p}_2,-}}{\varepsilon_{\mathbf{p}_1} + \varepsilon_{\mathbf{p}_2} - \varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}+\mathbf{p}_1+\mathbf{p}_2} + i0}. \end{aligned} \quad (29)$$

The T matrix is computed numerically, and Fig. 1 shows the resulting spin diffusion rate τ_F/τ_D in units of the Fermi frequency $1/\tau_F = E_F/\hbar$ as the solid (red) line: it is highest in the trap center and decreases proportional to the density in the outer regions. At unitarity, the scattering rate is about 10 times larger than at weak coupling $1/k_F a = -2$. In the Boltzmann regime, $\hbar/\tau_D = \frac{4\sqrt{2}n\lambda^3}{3\pi\beta} [1 - \beta\varepsilon_B - (\beta\varepsilon_B)^2 \exp(\beta\varepsilon_B) \text{Ei}(-\beta\varepsilon_B)]$ where $\text{Ei}(x)$ is the exponential integral and $\varepsilon_B = \hbar^2/ma^2$ [10]. At unitarity the scattering cross section decreases with temperature, and $\hbar/\tau_D = 4\sqrt{2}n\lambda^3/(3\pi\beta)$ [23].

Third, the mean field $\boldsymbol{\Omega}_{\text{mf}} = W\mathbf{m}(\mathbf{x}, t)/\hbar$ describes the precession of the spin current around the local magnetization and is given by a momentum average of the *real part* of the many-body T matrix $\mathcal{T}(\mathbf{p}_1, \mathbf{p}_2)$ over the momentum states between the majority and the minority Fermi surfaces, weighted by the velocity squared [10],

$$\begin{aligned} W &= \frac{1}{C_{\perp} |\mathbf{m}|} \int \frac{d^3 p_1}{(2\pi\hbar)^3} \frac{d^3 p_2}{(2\pi\hbar)^3} v_{1j}(v_{1j} - v_{2j})(n_{1+} - n_{1-}) \\ &\times (n_{2+} - n_{2-}) \text{Re } \mathcal{T}(\mathbf{p}_1, \mathbf{p}_2). \end{aligned} \quad (30)$$

At weak coupling $W = \mathcal{T}_0(0, 0) = 4\pi\hbar^2 a/m$ agrees with the bare interaction, which is real. At unitarity $1/a \rightarrow 0$, \mathcal{T}_0 becomes purely imaginary and W appears to vanish along with $\text{Re } \mathcal{T}_0$. This is indeed observed at high temperatures, but at low temperatures the *many-body* T matrix \mathcal{T} acquires a real part due to medium scattering, and $W \neq 0$ [4]. This is shown as the dash-dotted (blue) curve in Fig. 1: for weak coupling, $W \approx 4\pi\hbar^2 a/m$ is constant independent of the density and position in the trap, while at unitarity it decreases with density in the outer regions of the trap. Note that the very similar values for W at the trap center are coincidental: at weak coupling W is given essentially by the bare coupling, while at unitarity it is purely a many-body effect.

Spin rotation is characterized by the dimensionless Leggett-Rice parameter

$$\gamma = \mu n = -\frac{\tau_D W n}{\hbar} \quad (31)$$

which is plotted as the dashed (magenta) line in Fig. 1. At weak coupling,

$$\tau_D \propto \frac{1}{a^2 n^{4/3}}, \quad W \propto a, \quad \gamma \propto -\frac{1}{a n^{1/3}}, \quad (32)$$

hence γ becomes large and only weakly dependent on the density. At unitarity, on the other hand, γ is a purely many-body effect, much smaller, and roughly proportional to the density.

III. ANALYTICAL SOLUTIONS IN LIMITING CASES

The evolution equations (14)–(17) leave the density profile largely invariant, but the spin distribution changes dramatically as the magnetization is wound up into a helix according to the equations

$$\partial_t \mathbf{m} + \sum_j \nabla_j \mathbf{J}_j + \alpha x_3 \hat{\mathbf{z}} \times \mathbf{m} = 0 \quad (33)$$

$$\begin{aligned} \partial_t \mathbf{J}_j + \alpha_{\parallel} P_{\parallel} \nabla_j \mathbf{m} + \alpha_{\perp} P_{\perp} \nabla_j \mathbf{m} + \omega_j^2 x_j \mathbf{m} \\ + (\alpha x_3 \hat{\mathbf{z}} + \frac{W}{h} \mathbf{m}) \times \mathbf{J}_j = -\frac{\mathbf{J}_j}{\tau_D}. \end{aligned} \quad (34)$$

The full numerical solution of these equations is presented in Sec. IV. In order to gain a qualitative understanding of the spin evolution in a trapped gas, it is instructive to consider first the approximate analytical solutions in the homogeneous case.

If the scattering time τ_D is much shorter than any other relevant time scale, for instance, the dephasing time $\tau = (D^{\perp} \alpha^2)^{-1/3}$ due to the helix, the current reaches a steady state and its time derivative vanishes in the rotating frame, $\partial_t \mathbf{J}_j = -\alpha x_3 \hat{\mathbf{z}} \times \mathbf{J}_j$. Defining $D_0^{\perp} = \alpha_{\perp} \tau_D$ and $\mu \mathbf{m} = -\tau_D \Omega_{\text{mf}} = \gamma \mathbf{M}$ one finds

$$\mathbf{J}_j + D_0^{\perp} \nabla_j \mathbf{m} + \tau_D \omega_j^2 x_j \mathbf{m} - \mu \mathbf{m} \times \mathbf{J}_j = 0. \quad (35)$$

This equation is solved by the steady-state current

$$\begin{aligned} \mathbf{J}_j = -\frac{D_0^{\perp}}{1 + \mu^2 \mathbf{m}^2} \left\{ \nabla_j \mathbf{m} + \mu \mathbf{m} \times \nabla_j \mathbf{m} \right. \\ \left. + \mu \mathbf{m} (\mu \mathbf{m} \cdot \nabla_j \mathbf{m}) \right\} - \tau_D \omega_j^2 x_j \mathbf{m} \end{aligned} \quad (36)$$

where D_0^{\perp} , μ , and τ_D may still depend on position; the last term arises due to the trapping potential. Inserting this current into the continuity equation for the magnetization (33) yields

$$\begin{aligned} \partial_t \mathbf{m} = -\alpha x_3 \hat{\mathbf{z}} \times \mathbf{m} + \sum_j \omega_j^2 \nabla_j (\tau_D x_j \mathbf{m}) \\ + \nabla_j \frac{D_0^{\perp}}{1 + \mu^2 \mathbf{m}^2} \left\{ \nabla_j \mathbf{m} + \mu \mathbf{m} \times \nabla_j \mathbf{m} + \mu \mathbf{m} (\mu \mathbf{m} \cdot \nabla_j \mathbf{m}) \right\}. \end{aligned}$$

The Leggett solution [5] is recovered in the homogeneous limit $\omega_j = 0$, where \mathbf{m}^2 remains constant in space:

$$\partial_t \mathbf{m} = -\alpha x_3 \hat{\mathbf{z}} \times \mathbf{m} + \frac{D_0^{\perp}}{1 + \mu^2 \mathbf{m}^2} \{ \nabla^2 \mathbf{m} + \mu \mathbf{m} \times \nabla^2 \mathbf{m} \}.$$

In this case the longitudinal magnetization m_z remains unchanged, while the transverse magnetization $m_+ = m_x + im_y$ evolves as

$$\partial_t m_+ = -i\alpha x_3 m_+ + D_{\text{eff}}^{\perp} (1 + i\mu m_z) \nabla^2 m_+ \quad (37)$$

with effective diffusivity $D_{\text{eff}}^{\perp} = D_0^{\perp} / (1 + \mu^2 \mathbf{m}^2)$. Since \mathbf{m}^2 is constant in space, this is now a *linear* diffusion equation, albeit with a complex diffusion coefficient.

In the spin-echo protocol, the gradient α winds up the transverse magnetization $m_+ = m_x + im_y$ into a helix along the x_3 direction. A transverse spin current $J_{3+} \sim \partial_3 m_+$ appears, which tends to smooth the helix. If J_{3+} has a component perpendicular to the local magnetization $\mathbf{m} \approx m_z \hat{\mathbf{z}}$ (at small tipping angle) it precesses around it at frequency $\Omega_{\text{mf}} = W m_z / \hbar$. At time t_{π} , a π pulse around the y axis in Bloch space is applied; this is equivalent to reversing the sign of α . The subsequent time evolution unwinds the helix until the echo time $t_e = 2t_{\pi}$, where the transverse magnetization is again homogeneous. In the absence of spin rotation, $\gamma = 0$, the modulus of the transverse magnetization $A(t) = |m_+(t)|$ at time t_e decays as a cubic exponential [5]

$$A(t_e) = A_0 \exp\left(-\frac{D_0^{\perp} \alpha^2 t_e^3}{12}\right) \quad (38)$$

where $A_0 = A(0)$. This result is approximately correct even for $\gamma \neq 0$ if the tipping angle θ is small, $|m_+| \ll |m_z|$, and for short times. For finite γ , the magnetization decay is slowed down and is given by [4]

$$A(t_e) = A_0 \sqrt{\frac{1}{\eta} \mathcal{W}\left(\eta \exp\left[\eta - \frac{D_0^{\perp} \alpha^2 t_e^3}{6(1 + \gamma^2 M_z^2)}\right]\right)}, \quad (39)$$

$$\phi(t_e) = \gamma M_z \ln\left(\frac{A(t_e)}{A_0}\right), \quad (40)$$

where $\eta = \gamma^2 (A_0/n)^2 / (1 + \gamma^2 M_z^2)$ for polarization M_z , and $\mathcal{W}(z)$ is the Lambert- W function. This solution for the homogeneous system is used in the analysis of the experimental data to fit the diffusivity D_0^{\perp} and the apparent Leggett-Rice parameter γ from the measured magnetization decay and phase shift ϕ of the trapped system. In the following the full spin evolution in the trap is computed explicitly to determine to what extent the homogeneous solution is still applicable to the trapped gas.

IV. RESULTS

In the trapped gas, the assumptions which led to the analytical solutions in Sec. III (steady-state current, homogeneity) are not satisfied. Instead, a full numerical solution of the spin evolution equations, (33) and (34), is necessary. The initial condition in the experimental protocol [4] is a fully polarized cloud of fermionic atoms with a thermal profile, (19) and (22), for tipping angle θ . This distribution is stationary in the absence of an external magnetic field gradient α . Due to the density profile, (19), also the coefficients $\alpha_{\perp}(\mathbf{x})$, $\tau_{\perp}(\mathbf{x})$, and $W(\mathbf{x})$ depend on the position in the trap. For a small tipping angle $|m_+| \ll |m_z|$ the gas remains almost fully polarized, hence the density profile and the coefficients are time independent even in the presence of the gradient α .

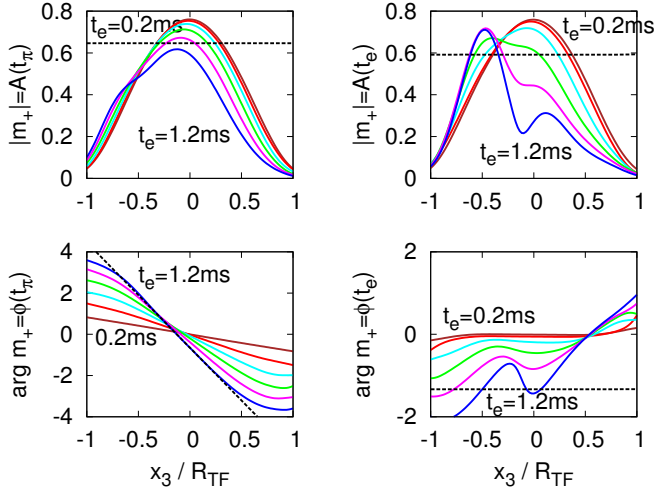


FIG. 2. (Color online) Transverse magnetization profiles on the BCS side $1/k_F a = -2$ at temperature $(T/T_F)_i = 0.2$ and times t_π just before the π pulse (left column), and at the echo time $t_e = 2t_\pi$ (right column). Upper panels display the amplitude $A = |m_+|$; lower panels, the phase angle $\phi = \arg m_+$. Solid curves in each panel are for a trapped gas at $t_e = 0.2$ ms to $t_e = 1.2$ ms, in comparison with a homogeneous gas at $t_e = 1.2$ ms (dashed line).

In the numerical solution, the experimental parameters [4] are used: gradient $\alpha = 1.67 \mu\text{m}^{-1} \text{kHz}$, trap frequency $\omega_3 = 2\pi \times 750$ Hz, $\bar{\omega} = 2\pi \times 470$ Hz, roughly $N \sim 40000$ atoms of ^{40}K with a Fermi energy of $E_F = 2\pi\hbar \times 29$ kHz, and Thomas-Fermi radius $R_{TF} = 5.1 \mu\text{m}$. Even though the Fermi gas is initially fully polarized, the Fermi wave vector $k_F = 12 \mu\text{m}^{-1}$ and the Fermi time $\tau_F = 0.0087$ ms are taken for a hypothetical balanced gas of the same atom number in order to recover the standard relation $n = k_F^3/3\pi^2$ for the total density [4]. At the lowest experimental temperature, $T \approx 280$ nK, the initial reduced temperature is $(T/T_F)_i = 0.2$ and $\lambda = 0.52 \mu\text{m}$. The phase-space density at the trap center reaches $\lambda^3 n = 7.3$, well within the quantum degenerate regime.

In Fig. 2, the helical state of the local magnetization in the trap at $1/k_F a = -2$ is shown for different times t_π (left column). At this time, the π pulse is applied (equivalently, the sign of α is reversed), and the subsequent time evolution unwinds the helix until the echo time $t_e = 2t_\pi$ (right column), which is chosen between $t_e = 0.2$ ms and $t_e = 1.2$ ms. For comparison, the density profiles for a homogeneous gas with $t_e = 1.2$ ms are shown as dashed lines. At time t_π , the ϕ profiles of the trapped and the homogeneous gas have a similar slope and are shifted toward negative phase angles by spin rotation. At time t_e , instead, the phase angle of the homogeneous gas is again homogeneous, but m_+ for the trapped gas remains in a helical state. Therefore, the trap-averaged transverse magnetization decays quickly even though the local m_+ is still sizable, and the apparent slope $\phi(t_e)/M_z \ln(A(t_e)/A_0)$ is lower than in the homogeneous case, where it reaches the microscopic

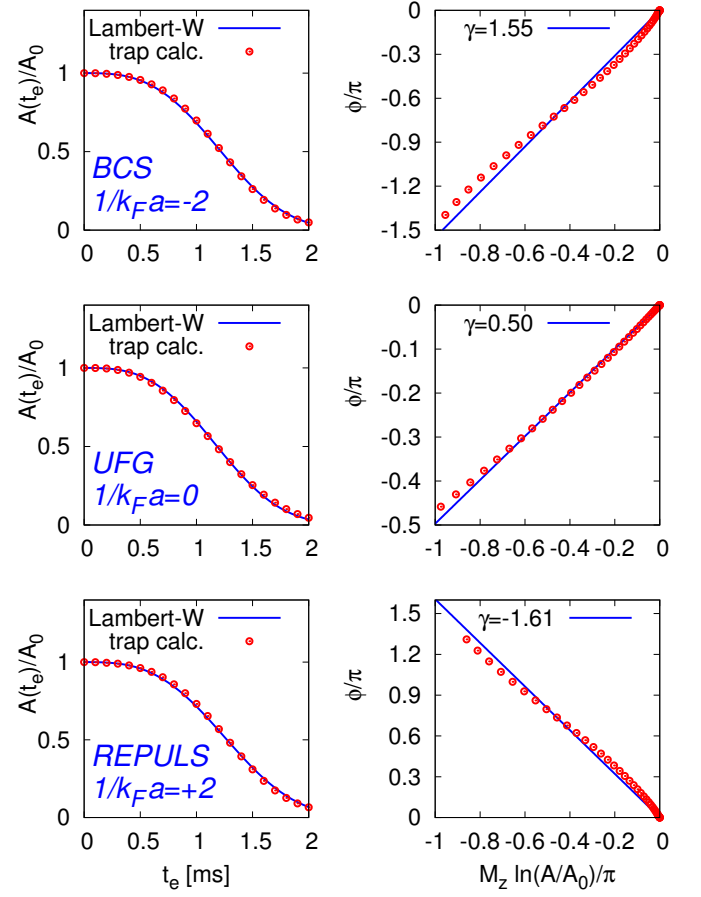


FIG. 3. (Color online) Amplitude and phase of the transverse magnetization m_+ at temperature $(T/T_F)_i = 0.2$ and small tipping angle $\theta = \pi/30$ ($M_z = 0.995$). Left: analytical solution, (39) (solid line), with D_0^\perp adjusted to fit the trap calculation (circles). Right: fit of γ from the slope of the phase shift ϕ in Eq. (40). Top, $1/k_F a = -2$ (BCS side); center, $1/k_F a = 0$ (unitarity); bottom, $1/k_F a = 2$ (repulsive branch).

value $\gamma = -W m_z \tau_D / \hbar$ according to Eq. (40). As we see below, this effect leads to a saturation of the apparent γ for weak coupling.

Note that the magnetization profiles are not symmetric with respect to x_3 : this is due to the spin-rotation term in Eq. (37) producing an additional phase shift on top of the gradient α . Consequently, if one reverses the sign of the scattering length a , the sign of γ is approximately reversed, and the resulting magnetization profile is the mirror image with $x_3 \mapsto -x_3$. Note also that at strong coupling $1/k_F a = 0$, the phase profile at t_e is nearly homogeneous at the trap center (not shown), and the kinetic coefficients for the trapped gas approach those in the homogeneous case.

Figure 3 shows the decay of the trap-averaged transverse magnetization with time (left column), and the growth of the phase angle ϕ with the slope of the magnetization decay (right column). From these plots, the apparent Leggett-Rice parameter $\gamma = \phi(t_e)/M_z \ln(A(t_e)/A_0)$

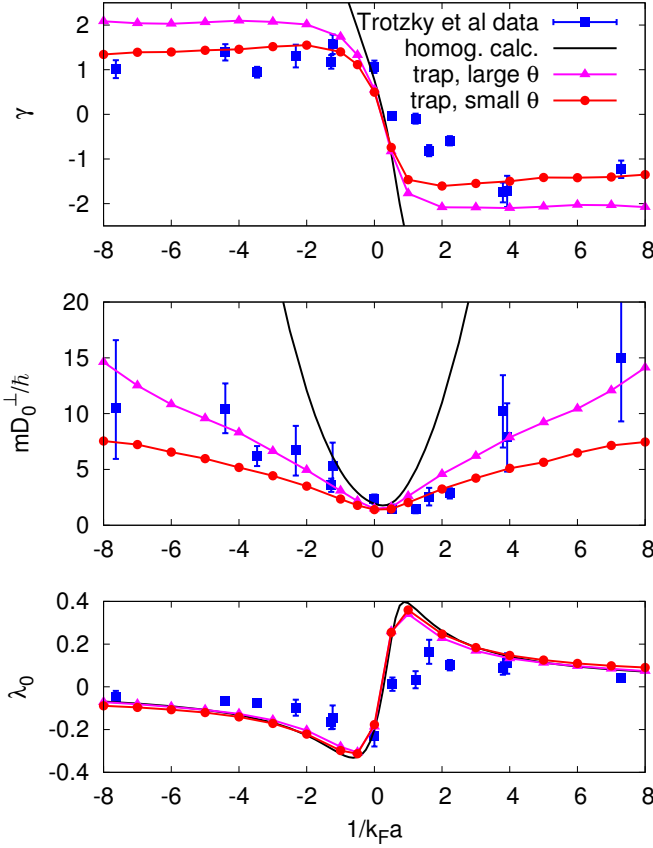


FIG. 4. (Color online) Interaction effect on spin transport: Leggett-Rice parameter γ , bare diffusivity D_0^\perp , and interaction parameter λ_0 as a function of $1/k_Fa$ at degenerate temperature $(T/T_F)_i = 0.2$. Results for small ($\theta = \pi/30$; circles) and large ($\theta = \pi/3$; triangles) tipping angles are shown and compared to the experimental data [4] (squares).

can be read off as the slope of the curve through the origin. This value of γ is then used to fit the magnetization decay on the left side of Fig. 3 to the analytical decay function (39) with diffusivity D_0^\perp . Here, I follow the analysis of the experimental data in Ref. 4 and fit the decay of the trapped gas to the homogeneous solution.

In Fig. 4, the apparent Leggett-Rice parameter γ extracted above is plotted versus the interaction strength, from the BCS regime (left) via unitarity ($1/k_Fa = 0$) to the repulsive branch (right). For a trapped gas, I find that the apparent γ saturates for weak coupling on both the BCS and the repulsive sides. This behavior differs qualitatively from the homogeneous case, where γ continues to increase linearly at weak coupling [cf. Eq. (32)] (solid black line). The trap calculation thus explains the saturation of γ observed in experiments [4]. The absolute value of γ for the trapped gas depends on the tipping angle θ of the initial polarization: for a large tipping angle $\theta = \pi/3$ as in the experiment, the Leggett-Rice parameter γ saturates at larger values than in the case of a small tipping angle $\theta = \pi/30$. The value to which γ saturates also depends on the strength of the gradient α , in contrast

to the homogeneous case (see below). Figure 4 also shows that the Leggett-Rice parameter changes sign, reflecting the sign of the effective interaction between quasiparticles which is attractive on the BCS side and repulsive on the repulsive branch [4]. The sign change occurs for slightly positive values of $1/k_Fa$.

The center panel in Fig. 4 shows the bare transverse diffusivity D_0^\perp . Again, the trap diffusivity is significantly lower than the homogeneous value (solid black line) at weak coupling and agrees with experiment. Once γ is known, the bare diffusivity D_0^\perp is found from the fit of the analytical solution (39) to the magnetization decay in Fig. 3.

The bottom panel in Fig. 4 displays the ratio

$$\lambda_0 = -\frac{\hbar\gamma}{2mD_0^\perp} = \frac{Wn}{2m\alpha_\perp}. \quad (41)$$

It measures the strength of the effective interaction irrespective of the scattering time τ_D and follows the sign change of γ since $D_0^\perp > 0$. Again, the trap calculation agrees with the experimental data except in the instability region $0 < 1/k_Fa \lesssim 1.3$ [4], while the homogeneous gas has more pronounced interaction effects.

Spin rotation in a trapped gas is determined qualitatively by the ratio between different length scales: the trap size $R_{TF} \approx 5\mu\text{m}$, the helix pitch $\ell_{\text{helix}} = 2\pi/(\alpha t_e/2) \approx 4\mu\text{m}$ for $t_e = 2\text{ms}$, and the mean free path $\ell_{\text{mfp}} = v_F\tau_D$, which ranges from $0.5\mu\text{m}$ at unitarity to $3.8\mu\text{m}$ at $1/k_Fa = -2$. In simulations with a gradient α larger than the experimental value, several helix pitches fit into the trap, $R_{TF} \gg \ell_{\text{helix}}$, and the trap averaged diffusivity and γ are equal to their homogeneous values as long as also $\ell_{\text{mfp}} < \ell_{\text{helix}}$, but saturate for larger ℓ_{mfp} . This is in marked contrast to longitudinal spin diffusion, where a scaling factor of about 5 was found to relate the trap-averaged and homogeneous diffusivity D_\parallel [17, 23, 24]. For weaker gradients where $\ell_{\text{helix}} > R_{TF}$, less than one helix pitch fits into the trap and the homogeneous solution is reached not even at unitarity where $R_{TF} \gg \ell_{\text{mfp}}$.

Finally, Fig. 5 shows the temperature dependence of the Leggett-Rice parameter at unitarity. γ decreases with temperature, which is understood as follows: the two-body scattering amplitude is purely imaginary at unitarity and would imply a vanishing $\gamma \sim \text{Re}\mathcal{T}$. Hence, the observed finite value of γ in the quantum degenerate regime $(T/T_F)_i < 1$ is a *many-body* effect due to medium scattering. The presence of the medium enhances both the dissipative and the reactive effects of scattering at low temperatures, and more so in the homogeneous gas than in the trap, where only the core is strongly interacting.

Similarly, the diffusivity reaches values of $D_0^\perp \simeq 2\hbar/m$ in the quantum degenerate regime (center panel), in agreement with experiment, while the ratio λ_0 becomes large only at the lowest temperatures (lower panel).

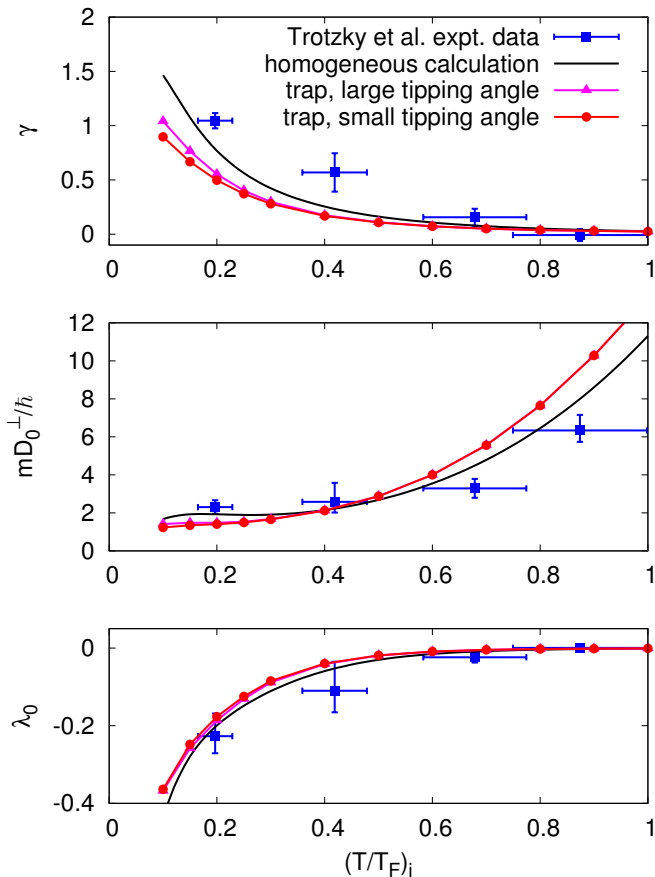


FIG. 5. (Color online) Temperature effect on spin transport at unitarity: Leggett-Rice parameter γ , bare diffusivity D_0^\perp , and interaction parameter λ_0 as a function of the initial temperature $(T/T_F)_i$. At unitarity the results for small ($\theta = \pi/30$; circles) and large ($\theta = \pi/3$; triangles) tipping angles differ very little; (blue) squares are experimental data [4].

V. CONCLUSION

To summarize, a spin-echo sequence in a trapped Fermi gas is modeled by a nonlinear and complex diffusion equation for the transverse magnetization. The spin evolution exhibits the Leggett-Rice effect of strength $\gamma = \mu n$, which appears to saturate for weak coupling, and a bare diffu-

sivity much lower than expected for a homogeneous gas of the same temperature and interaction strength. These results are obtained without any fit parameters and agree very well with the weak-coupling data measured recently for a trapped gas of ultracold fermionic ^{40}K atoms [4]. The present calculation provides an intuitive interpretation of the observed saturation of γ : while the spin helix in the homogeneous gas is completely unwound at the echo time t_e , the trapped gas remains partially in a helical state, with the average transverse magnetization $A(t_e)$ strongly reduced and a smaller phase shift $\phi(t_e)$. At weak coupling $1/|k_F a| \gtrsim 2$ the kinetic theory employed in this study is well controlled and includes the relevant interaction effects.

At strong coupling $1/k_F a = 0$ (unitarity), the Leggett-Rice effect is absent at the two-body level and arises only due to many-body medium scattering. The trap calculation at strong coupling agrees with the experimental results qualitatively; the remaining differences may be due to reheating in a demagnetized Fermi gas [3, 4] or due to interaction corrections to the equation of state which were not included in this study; they remain a topic for future work.

Ultracold Fermi gases are thus ideal systems to study the interaction and temperature dependence of the Leggett-Rice effect and the spin transport coefficients. The spin dynamics of the trapped gas (Fig. 2) is much more complex than in the homogeneous case and requires a numerical solution. The spin evolution simplifies near unitarity if $R_{\text{TF}} > \ell_{\text{helix}} > \ell_{\text{mfp}}$, in which case the homogeneous solution is recovered without any trap-related scaling factors. The decay of the trap-averaged magnetization can be analyzed by the methods developed for the homogeneous case: the magnetization decay fits the homogeneous solution surprisingly well (Fig. 3). This study shows that the extracted transport coefficients for the trapped gas can differ markedly, especially at weak coupling, from those of the corresponding homogeneous system (Fig. 4). Conveniently for the interpretation, the sign of γ , which reveals the repulsive or attractive character of the effective interaction, does not change with the trap average.

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